Brightness and Distance

What is the relationship between apparent brightness and distance? What *is* brightness, anyway?

Let's start by discussing the *intrinsic* brightness of a star. Astronomers like to use the term **luminosity** to describe the amount of energy the star radiates in all directions per unit time. **Visual luminosity** means the luminosity at visual wavelengths, that is wavelengths that humans can see. **Bolometric luminosity** is the luminosity at *all* wavelengths of the electromagnetic spectrum. The bolometric luminosity of the Sun is 3.85×10^{26} watts. A watt is one joule of energy per second.

When we look at a star, though, we are intercepting only a tiny fraction of the light that it emits. The amount of light that we see is called the **luminous intensity**. This luminous intensity is measured in candelas. A candela is one lumen per steradian (a unit of solid angle). It is important to note that luminous intensity takes into account the response of the human visual system. In other words, our eyes are not equally sensitive to all wavelengths of visible light.

Since the response of the human eye to brightness is close to logarithmic (it is actually a power law), astronomers traditionally use the **magnitude system** to describe the luminous intensity of a star. Since stars are at different distances, the only way to compare their intrinsic magnitudes or brightness is to define a **standard distance**. That standard distance is arbitrarily defined to be 10 parsecs. The apparent magnitude a star would have at the standard distance of 10 pc is called the **absolute magnitude**. Apparent magnitude is the magnitude that we observe, corrected for atmospheric extinction. The apparent magnitude of a star at visual wavelengths is called the apparent visual magnitude, and is denoted by the symbol m_v. The absolute visual magnitude is denoted by the symbol M_v.

What is the mathematical relationship between magnitude and luminous intensity?

$$(m_1 - m_2) = 2.5 \log\left(\frac{i_2}{i_1}\right)$$

where m_1 is the magnitude of star #1
  and m_2 is the magnitude of star #2
  and i_1 is the luminous intensity of star #1
  and i_2 is the luminous intensity of star #2

But what's a log? A logarithm is an exponent, and a way to represent a large range of numbers with smaller numbers.

$10^1 = 10$	$\log 10 = 1$	$10^{-1} = 0.1$	$\log 0.1 = -1$
$10^2 = 100$	$\log 100 = 2$	$10^{-2} = 0.01$	$\log 0.01 = -2$
$10^3 = 1{,}000$	$\log 1000 = 3$	$10^{-3} = 0.001$	$\log 0.001 = -3$
$10^4 = 10{,}000$	$\log 10000 = 4$	$10^{-4} = 0.0001$	$\log 0.0001 = -4$
$10^5 = 100{,}000$	$\log 100000 = 5$	$10^{-5} = 0.00001$	$\log 0.00001 = -5$
$10^6 = 1{,}000{,}000$	$\log 1000000 = 6$	$10^{-6} = 0.000001$	$\log 0.000001 = -6$

In case you were wondering, $10^0 = 1$, so $\log 1 = 0$.

$\log 10 = 1$	$10^1 = 10$
$\log 20 \approx 1.30$	$10^{1.30} \approx 20$
$\log 30 \approx 1.48$	$10^{1.48} \approx 30$
$\log 40 \approx 1.60$	$10^{1.60} \approx 40$
$\log 50 \approx 1.70$	$10^{1.70} \approx 50$
$\log 60 \approx 1.78$	$10^{1.78} \approx 60$
$\log 70 \approx 1.85$	$10^{1.85} \approx 70$
$\log 80 \approx 1.90$	$10^{1.90} \approx 80$
$\log 90 \approx 1.95$	$10^{1.95} \approx 90$
$\log 100 = 2$	$10^2 = 100$

Now, getting back to our logarithmic equation relating magnitude to luminous intensity…

$$(m_1 - m_2) = 2.5 \log\left(\frac{i_2}{i_1}\right)$$

If star #2 is 100 times as bright as star #1, what is their difference in magnitude? Which star has the lower-number magnitude?

Now, algebraically manipulate the above equation so that $i_2/i_1 = f(m_1 - m_2)$. Then, calculate the apparent luminous intensity ratio of Sirius, m_v –1.44, compared to Polaris, $m_v = 1.97$.

Finally, here is the equation relating absolute and apparent magnitude to distance.

$$M = m + 5\,(1 - \log d)$$

where M = the absolute magnitude of a star
and m = the apparent magnitude of a star
and d = the distance to the star in parsecs

> Derive the equation for m, given M and d.
>
> Then, derive the equation for d, given M and m.

Most of us can see stars no fainter than about $m_v = +6.0$, given light pollution and our aging eyes. How far away (in light years) would we have to travel before the Sun would be just barely visible to the unaided eye? The absolute visual magnitude of the Sun, $M_v = 4.82$. 1 pc = 3.26 ly.

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