How Long is That Star Above the Horizon?

Have you ever wondered how long a star or planet is above the horizon? Basically, that depends on the declination of the celestial object and your latitude.

In the northern hemisphere,

If $\delta > (90^\circ - \varphi)$, then the object is always above your horizon (circumpolar)

where δ = the object's declination in decimal degrees

and φ = your latitude in decimal degrees

If $\delta < (\varphi - 90^\circ)$, then the object is always below your horizon (celopolar)

And, if $(\varphi - 90^\circ) \leq \delta \leq (90^\circ - \varphi)$, then the object is above the horizon some of the day and below the horizon some of the day.

For example, Ridgeway, Wisconsin has a latitude of exactly 43° and a longitude of exactly 90°. To calculate the range of declinations where objects rise and set each day, the longitude is irrelevant but the latitude is important. We get (90° - 43°) = +47° and (43° - 90°) = -47° for our boundaries, so the range of rising/setting declinations is +47° to -47°. By the way, the declination of the point directly overhead (the zenith) is always equal to your latitude—in this case, +43°.

Now, how long is a celestial object above the horizon? We'll do the calculation in two steps.

1: $$H = \cos^{-1}\left(\frac{-\sin\varphi\sin\delta}{\cos\varphi\cos\delta}\right)$$

where H = is an angle in degrees
and φ = your latitude
and δ = the object's declination

2: $$T = \left(\frac{H}{7.42}\right)$$

where T = the number of hours the object is above the horizon each day

In step 2 on the previous page, I've included a "fudge factor" that accounts for the fact that starrise to starrise is $23^h56^m04^s$ instead of 24^h (the difference between the **sidereal day** and the **mean solar day**), and also that atmospheric refraction makes an object appear to rise earlier and set later than it really does.

The equations on the previous page give a result that is generally accurate to about ±1 minute. Let's try an example.

Location: Ridgeway, Wisconsin
Latitude: 43° N

Star: Arcturus
Declination: +19.153°

$$H = \cos^{-1}\left(\frac{-\sin 43^\circ \sin 19.153^\circ}{\cos 43^\circ \cos 19.153^\circ}\right) = 108.898^\circ \qquad T = \left(\frac{108.898^\circ}{7.42}\right) = 14.68^h = 14^h41^m$$

So, at Ridgeway, Wisconsin, Arcturus is above the horizon 14 hours and 41 minutes each day.

Now, what happens if you put in a circumpolar or celopolar declination? Let's use +50° and -50° to find out...

$$H = \cos^{-1}\left(\frac{-\sin 43^\circ \sin 50^\circ}{\cos 43^\circ \cos 50^\circ}\right) = \cos^{-1}(-1.11) = \text{undefined!}$$

$$H = \cos^{-1}\left(\frac{-\sin 43^\circ \sin(-50^\circ)}{\cos 43^\circ \cos(-50^\circ)}\right) = \cos^{-1}(1.11) = \text{undefined!}$$

In the extreme example, using either +90° or -90° declination, we end up with a cos(90˚) or cos(-90˚) in the denominator, which equals zero, and dividing by zero gives us $\cos^{-1}(\infty)$, which is also undefined!

Finally, let me say a word about my proposed term **celopolar** (pronounced KELL-o-POLE-ar). Since "circum" comes from the Latin word meaning "around", I'd like to suggest we use the Latin word "celo" meaning "to hide" for stars that never rise above the horizon. Since "circumpolar" means literally "around the pole", "celopolar" would mean "to hide from the pole", in other words, an object that is always below the horizon.

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